First Semester Examination
Academic Session 2018/2019

December 2018/January 2019

MST564 – Statistical Reliability
(Kebolehpercayaan Statistik)

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of TWELVE (12) pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi DUA BELAS (12) muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **SIX (6)** questions.

*[**Arahan :** Jawab **ENAM (6)** soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai].

Question 1

Consider the three-parameter distribution with hazard function,

$$h(t) = \alpha + \frac{\beta}{t+\gamma}.$$

(a) Examine the range of values that α, β, and γ can take. Investigate $h(t)$ and show that it can be monotone increasing or monotone decreasing, according to the values of the parameters.

(b) Determine the probability density function and survival function for the distribution.

[15 marks]

Soalan 1

Pertimbangkan suatu taburan yang mengandungi tiga parameter dengan fungsi bahaya,

$$h(t) = \alpha + \frac{\beta}{t+\gamma}.$$

(a) Periksa julat nilai yang mana α, β, dan γ boleh diambil. Periksa $h(t)$ dan tunjukkan bahawa ia boleh menjadi meningkat secara monotonik atau menurun secara monotonik dengan mengikut nilai-nilai parameter.

(b) Tentukan fungsi ketumpatan kebarangkalian dan fungsi mandiri bagi taburan tersebut.

[15 markah]

Question 2

The lifetime of a manufactured item is thought to be exponentially distributed. n items are selected and put on test until r of them fail at times $t_{(1)}, t_{(2)}, \ldots, t_{(r)}$.

(a) Write down the likelihood function for the experiment and derive the maximum likelihood estimator for the mean. State (without proof) the sampling distribution of this estimator and explain how you would use it to obtain a confidence interval for the mean lifetime.

(b) In an experiment with $n = 10$ days, it was found that seven items failed with failure times 2, 4, 14, 24, 27, 33 and 51 days respectively. Obtain a 95% confidence interval for the mean lifetime of an item.

[15 marks]

Soalan 2

Jangka masa hayat bagi sesuatu barang yang diproduksi dianggap bertabur secara eksponen. n item dipilih dan diuji sehingga r daripada mereka gagal pada masa $t_{(1)}, t_{(2)}, \ldots, t_{(r)}$.

(a) *Tuliskan fungsi kebolehjadian untuk eksperimen ini dan dapatkan penganggar kebolehjadian maksimum bagi min. Nyatakan (tanpa bukti) taburan pensampelan untuk penganggar ini dan jelaskan bagaimana anda akan menggunakannya untuk mendapatkan selang keyakinan untuk min jangka hayat.*

(b) *Dalam percubaan dalam 10 hari, didapati tujuh item gagal pada masa kegagalan 2, 4, 14, 24, 27, 33 dan 51 hari. Dapatkan selang keyakinan 95% untuk min jangka hayat bagi barang itu.*

[15 markah]

Question 3

The data below show survival times (in months) of patients with Hodgkin's disease who were treated with nitrogen mustards. Group A patients received little or no prior therapy, whereas Group B patients received heavy prior therapy. Observations with asterisks are censoring times.

Group A	1.25	1.41	4.98	5.25	5.38
	6.92	8.89	10.98	11.18	13.11
	13.21	16.33	19.77	21.08	21.84*
	22.07	31.38*	32.62*	37.18*	42.92
Group B	1.05	2.92	3.61	4.2	4.49
	6.72	7.31	9.08	9.11	14.49*
	16.85	18.82*	26.59*	30.26*	41.34*

(a) Obtain and compare Kaplan-Meier estimates for the two groups. Does there appear to be a difference in the 1 year survival probability for the two types of patients?

(b) Determine confidence limits for $S(1)$ and for the median survival time $t_{0.5}$ for each group.

[15 marks]

Soalan 3

Data di bawah menunjukkan jangka masa hayat (dalam bulan) pesakit dengan penyakit Hodgkin yang dirawat dengan mustard nitrogen. Pesakit Kumpulan A menerima sedikit atau tiada terapi terdahulu, manakala pesakit Kumpulan B mendapat terapi terdahulu. Pemerhatian berbintang adalah masa penapisan.

Kumpulan A	*1.25*	*1.41*	*4.98*	*5.25*	*5.38*
	6.92	*8.89*	*10.98*	*11.18*	*13.11*
	13.21	*16.33*	*19.77*	*21.08*	*21.84**
	22.07	*31.38**	*32.62**	*37.18**	*42.92*
Kumpulan B	*1.05*	*2.92*	*3.61*	*4.2*	*4.49*
	6.72	*7.31*	*9.08*	*9.11*	*14.49**
	16.85	*18.82**	*26.59**	*30.26**	*41.34**

(a) Dapatkan dan bandingkan anggaran Kaplan-Meier untuk kedua-dua kumpulan. Adakah terdapat perbezaan dalam kebarangkalian mandiri dalam 1 tahun untuk kedua-dua jenis pesakit?

(b) Tentukan had keyakinan untuk $S(1)$ dan bagi masa hidup median bagi setiap kumpulan.

[15 markah]

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Question 4

The life time data in Table 1 are from a study involving 112 patients with plasma cell myeloma treated at National Cancer Institute. Use plots of empirical estimates of the survivor and hazard functions to suggest possible models.

Table 1: Survival Times for Patients with Plasma Cell Myeloma

Interval (Months)	Number of Risk at Start	Number of Withdrawals
[0,5.5)	112	1
[5.5,10.5)	93	1
[10.5,15.5)	76	3
[15.5,20.5)	55	0
[20.5,25.5)	45	0
[25.5,30.5)	34	1
[30.5,40.5)	25	2
[40.5,50.5)	10	3
[50.5,60.5)	3	2
[60.5, ∞)	0	0

[15 marks]

Soalan 4

Data jangka hayat dalam Jadual 1 adalah dari kajian yang melibatkan 112 pesakit dengan myeloma sel plasma yang dirawat di Institut Kanser Kebangsaan. Gunakan plot anggaran empirikal bagi mangsa yang hidup dan fungsi bahaya untuk mencadangkan model yang mungkin.

Jadual 1: Masa Hayat bagi Pesakit dengan Myeloma Sel Plasma

Selang (Bulan)	*Bilangan Risiko pada Permulaan*	*Bilangan Pengeluaran*
[0,5.5)	112	1
[5.5,10.5)	93	1
[10.5,15.5)	76	3
[15.5,20.5)	55	0
[20.5,25.5)	45	0
[25.5,30.5)	34	1
[30.5,40.5)	25	2
[40.5,50.5)	10	3
[50.5,60.5)	3	2
[60.5, ∞)	0	0

[15 markah]

Question 5

A subsystem consists of two assemblies A and B, connected in series. Assembly B consists of two components B_1 and B_2, connected in parallel. Assume that components operate independently. Components are purchased from one of three suppliers. Reliabilities of the three components from suppliers are:

	Supplier		
Component	1	2	3
A	0.99	0.92	0.97
B_1	0.85	0.90	0.93
B_2	0.90	0.99	0.82

(a) Draw a block diagram of the subsystem.

(b) Determine which supplier should be selected as the source for the three components.

(c) If the purchase policy were changed so that components could be bought individually, what would be the reliability of the best subsystem design?

[15 marks]

Soalan 5

Suatu subsistem terdiri daripada dua pemasang A dan B, disambungkan secara bersiri. Pemasang B terdiri daripada dua komponen, B_1 dan B_2, bersambung selari. Anggapkan bahawa komponen beroperasi secara bebas. Komponen dibeli daripada salah satu daripada tiga pembekal. Kebolehcapaian ketiga komponen dari pembekal adalah:

	Pembekal		
Komponen	*1*	*2*	*3*
A	*0.99*	*0.92*	*0.97*
B_1	*0.85*	*0.90*	*0.93*
B_2	*0.90*	*0.99*	*0.82*

(a) Lukis gambarajah blok subsistem.

(b) *Tentukan pembekal mana yang harus dipilih sebagai sumber bagi ketiga-tiga komponen tersebut.*

(c) *Jika dasar pembelian telah diubah supaya komponen boleh dibeli secara individu, apakah kebolehpercayaan reka bentuk subsistem terbaik?*

[15 markah]

Question 6

The data in Table 2 (use the data file provided: Q6tbl2.mtw) are from a more comprehensive set given by Krall et al. (1975). The problem is to relate survival times for multiple myeloma patients to a number of prognostic variables. The data given here show survival times, in months, for 65 patients and include measurements on each patient for the following five covariates:

x_1 Logarithm of a blood urea nitrogen measurement at diagnosis

x_2 Hemoglobin measurement at diagnosis

x_3 Age at diagnosis

x_4 Sex: 0, male; 1, female

x_5 Serum calcium measurement at diagnosis

Asterisks denote censoring times.

(a) Perform an appropriate analysis of the data.

(b) Interpret your results obtained in (a).

Table 2: Survival Times and Covariates for Multiple Myeloma Patients

t	x_1	x_2	x_3	x_4	x_5
1	2.218	9.4	67	0	10
1	1.94	12.0	38	0	18
⋮	⋮	⋮	⋮	⋮	⋮
57*	1.255	12.5	66	0	11
77*	1.079	14.0	60	0	12

[25 marks]

Soalan 6

Data dalam Jadual 3 (gunakan fail data yang disediakan: Q6tbl2.mtw) adalah dari set yang lebih komprehensif yang diberikan oleh Krall et al. (1975). Masalahnya ialah untuk mengaitkan masa hayat untuk pelbagai pesakit myeloma kepada beberapa pembolehubah prognostik. Data yang diberikan di sini menunjukkan masa survivor, dalam bulan, untuk 65 pesakit dan termasuk pengukuran pada setiap pesakit untuk lima kovariat berikut:

x_1 *Logaritma pengukuran nitrogen urea darah pada diagnosis*

x_2 *Pengukuran hemoglobin pada diagnosis*

x_3 *Umur pada diagnosis*

x_4 *Seks: 0, lelaki; 1, perempuan*

x_5 *Pengukuran kalsium serum pada diagnosis*

Asterisk menunjukkan masa penapisan.

(a) *Lakukan analisa data yang sesuai.*

(b) *Huraikan keputusan anda yang diperolehi dalam (a).*

Jadual 2: Masa Hayat dan Kovariat untuk Pelbagai Pesakit Myeloma

t	x_1	x_2	x_3	x_4	x_5
1	2.218	9.4	67	0	10
1	1.94	12.0	38	0	18
⋮	⋮	⋮	⋮	⋮	⋮
57*	1.255	12.5	66	0	11
77*	1.079	14.0	60	0	12

[25 markah]

APPENDIX / *APENDIKS*

General Formulas of Differentiation/*Formula Umum bagi Pembezaan*

$$\frac{d}{dt}c = 0$$

$$\frac{d}{dt}\left[cf(t)\right] = cf'(t)$$

$$\frac{d}{dt}\left[f(t)+g(t)\right] = f'(t)+g'(t)$$

$$\frac{d}{dt}\left[f(t)g(t)\right] = f(t)g'(t)+g(t)f'(t)$$

$$\frac{d}{dt}\left[\frac{f(t)}{g(t)}\right] = \frac{g(t)f'(t)-f(t)g'(t)}{\left[g(t)\right]^2}$$

$$\frac{d}{dt}f\left(g(t)\right) = f'\left(g(t)\right)g'(t)$$

$$\frac{d}{dt}t^n = nt^{n-1}$$

$$\frac{d}{dt}\left(e^t\right) = e^t$$

$$\frac{d}{dt}\left(a^t\right) = a^t \ln a$$

$$\frac{d}{dt}\ln|t| = \frac{1}{t}$$

General Formulas of Integrals/*Formula Umum bagi Pengamiran*

$$\int u\,dv = uv - \int v\,du$$

$$\int u^n du = \frac{1}{n+1}u^{n+1} + c,\ n \neq -1$$

$$\int \frac{du}{u} = \ln|u| + c$$

$$\int e^u du = e^u + c$$

$$\int a^u du = \frac{1}{\ln a}a^u + c$$

Formulas of Reliability / *Formula Kebolehpercayaan*

$$F(t) = \int_0^t f(x)dx$$

$$R(t) = 1 - F(t)$$

$$f(t) = \frac{dF(t)}{dt} = -\frac{dR(t)}{dt}$$

$$h(t) = \frac{f(t)}{R(t)}$$

$$H(t) = \int_0^t h(x)dx$$

$$R(t) = e^{-H(t)}$$

$$H(t) = -\ln R(t)$$

$$MTTF = \int_0^\infty tf(t)dt = \int_0^\infty R(t)dt$$

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Lifetime following an Exponential Distribution: / ***Masa hayat mengikut Taburan Eksponen:***

$$f(t) = \lambda e^{-\lambda t}$$

$$F(t) = 1 - e^{-\lambda t}$$

$$R(t) = e^{-\lambda t}$$

$$h(t) = \lambda$$

$$H(t) = \lambda t$$

$$MTTF = \frac{1}{\lambda}$$

Lifetime following a Weibull Distribution: / ***Masa hayat mengikut Taburan Weibull:***

$$f(t) = \beta \alpha^{-\beta} t^{\beta-1} \exp\left[-\left(\frac{t}{\alpha}\right)^{\beta}\right]$$

$$F(t) = 1 - e^{-\left(\frac{t}{\alpha}\right)^{\beta}}$$

$$R(t) = e^{-\left(\frac{t}{\alpha}\right)^{\beta}}$$

$$h(t) = \beta \alpha^{-\beta} t^{\beta-1}$$

$$H(t) = \left(\frac{t}{\alpha}\right)^{\beta}$$

$$MTTF = \alpha \Gamma\left(1 + \frac{1}{\beta}\right)$$

Lifetime following a Normal Distribution: / ***Masa hayat mengikut Taburan Normal:***

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2} \frac{(t-\mu)^2}{\sigma^2} \right]$$

$$F(t) = \Phi\left(\frac{t-\mu}{\sigma} \right)$$

$$R(t) = 1 - \Phi\left(\frac{t-\mu}{\sigma} \right)$$

$$h(t) = \frac{f(t)}{1 - \Phi\left(\frac{t-\mu}{\sigma} \right)}$$

Lifetime following a Lognormal Distribution: / ***Masa hayat mengikut Taburan Lognormal:***

$$f(t) = \frac{1}{\sqrt{2\pi}st} \exp\left[-\frac{1}{2s^2} \left(\ln \frac{t}{t_{median}} \right)^2 \right]$$

$$F(t) = \Phi\left(\frac{1}{s} \ln \frac{t}{t_{median}} \right)$$

$$R(t) = 1 - \Phi\left(\frac{1}{s} \ln \frac{t}{t_{median}} \right)$$

$$MTTF = t_{median} \exp\left(\frac{s^2}{2} \right)$$

$$t_R = t_{median} \exp\left(s z_{1-R} \right)$$

- ooo00ooo -